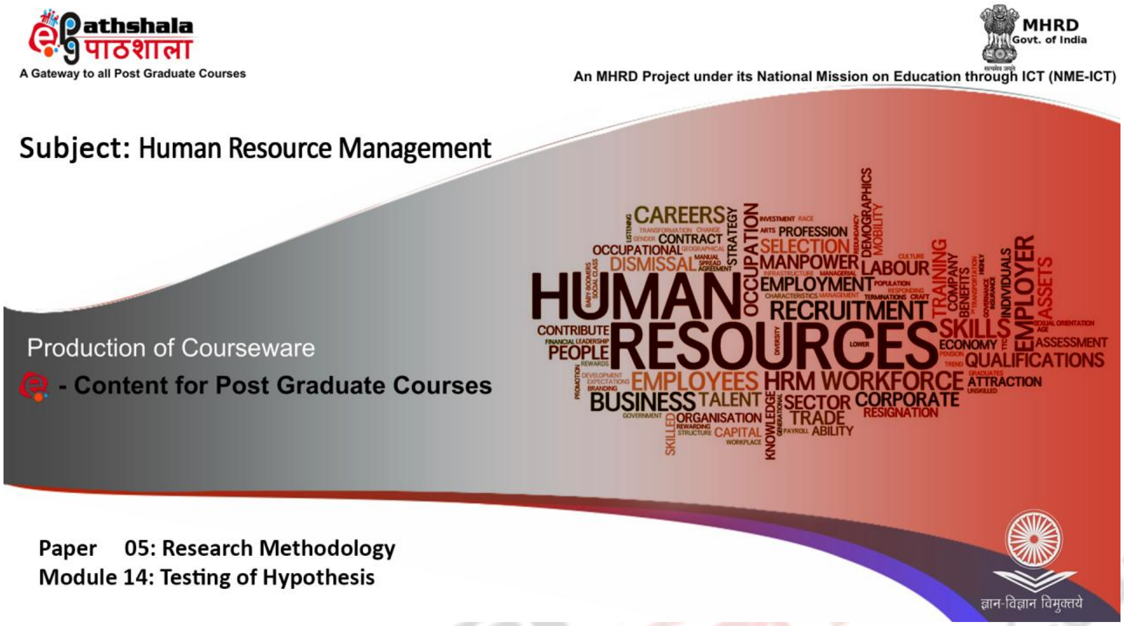


Development Team

Principal Investigator:	Prof. Ipshita Bansal Department of Management Studies BPS Women University, Khanpur Kalan, Sonepat
Paper Coordinator:	Prof. S.P. Singh Faculty of Management Studies Gurukul Kangri University, Haridwar
Content Writer:	Prof. S.P. Singh Faculty of Management Studies Gurukul Kangri University, Haridwar
Content Reviewer:	Prof. H.L. Verma Vice-Chancellor Jagannath University, Bahadurgarh, Haryana

Items	**Description of Module**
Subject Name	Management
Paper Name	Research Methodology
Module Title	**TESTING OF HYPOTHESES**
Module ID	Module 14
Pre-Requisites	Understanding the nature of hypotheses and tests
Objectives	To study the testing of hypotheses
Keywords	Null hypotheses, Alternative hypotheses, Mean, Proportion, Population

Role	Name	Affiliation
Principal Investigator	Prof.Ipshita Bansal	Department of Management Studies, BPSMV, Khanpur Kalan, Sonipat
Co-Principal Investigator		
Paper Coordinator	Prof. S.P.Singh	Department of Management Studies, GKV, Haridwar
Content Writer (CW)	Prof. S.P.Singh	Department of Management Studies, GKV, Haridwar
Content Reviewer (CR)		
Language Editor (LE)		

QUADRANT –I

1. Module 14 : Testing of Hypotheses
2. Learning Outcome
3. Tests and Errors
4. Steps in Testing of Hypotheses
5. Test statistics
Summary

1. **Module: Testing of Hypotheses**
2. **LEARNING OUTCOME:**

   After studying this module, you shall be able to

   - Know the tests and errors
   - Understand the steps in testing of hypotheses
   - Comprehend the test statistics

3. **Introduction**

   The major research objective of inferential statistics is testing of research hypothesis through statistical hypotheses. The hypothesis is tested based on the information obtained from a sample. Business uses the hypothesis tests widely and industry uses them for decision-making. There are two types of hypotheses: Null hypothesis and alternative hypothesis. Similarly, there are two tests: one tailed test and two-tailed tests.

4. **Statistical Significance**

Following the sampling approach, a hypothesis is accepted or rejected on the basis of sampling information along. Since any sample will almost vary somewhat from its population, it must be judged whether these differences are statistically significant. A difference has statistical significance if there is a reason to believe the difference does not represent random sampling fluctuations only.

In tests of significance, two kinds of hypothesis are used. The null hypothesis is used for testing. Null hypothesis is a statement that no difference exists between the parameter and the statistic being compared to it. The researchers usually test to determine whether a real difference exists. The hypothesis for testing is not stated in a positive form. This type of hypothesis cannot be tested definitely. Evidence that is consistent with a hypothesis stated in a positive form can almost never be taken as conclusive grounds for accepting the hypothesis. A finding consistent with this type of hypothesis might be consistent with other kinds of hypothesis also, and thus does not demonstrate the truth of the given hypothesis.

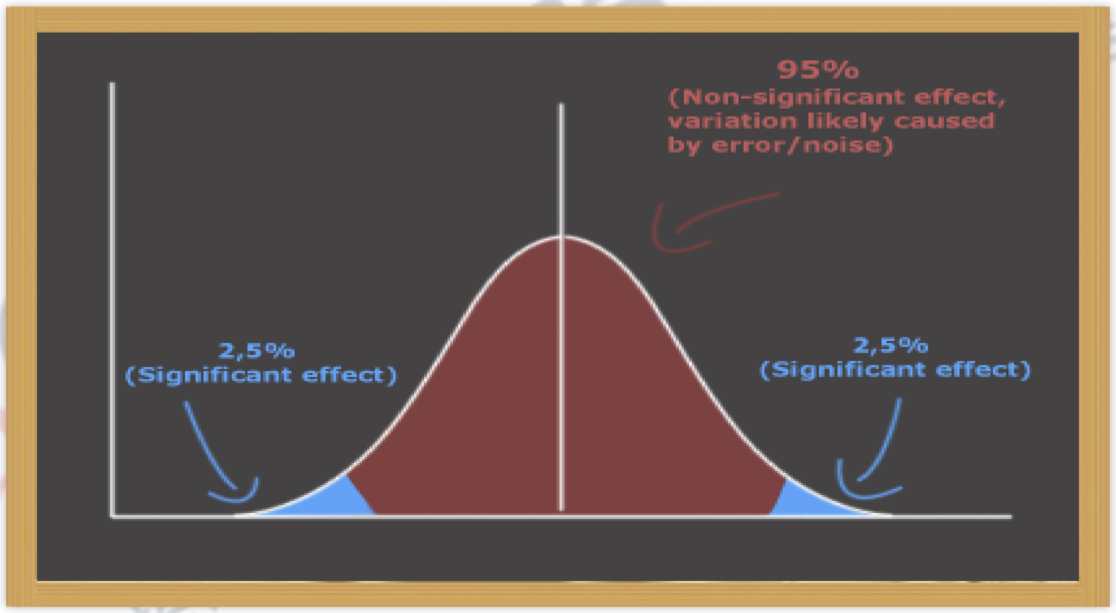


Figure 1 Statistical Significance
Adapted from: sites.google.com

5. **One tailed and two-tailed tests**

A test is called one-tailed only if the null hypothesis gets rejected when the value of the test statistic falls in acceptance region of the distribution.

A test is regarded as one-tailed if the null hypothesis is rejected at the time the value of the test statistic occurs in the region of acceptance of the distribution.

The test is two-tailed if null hypothesis gets rejected when the value of the test statistic occurs in either of the two tails of its sampling distribution.

The test is considered as two-tailed if the null hypothesis is rejected at the time the value of the test statistic occurs in the either of the two tails of the sampling distribution

For example, a cold-drink bottling plant distribution soft drinks in bottles of 300 ml maximum amount of liquid through a self-operated plant. The company incurs huge amount of loss in case if bottles are overfilled. It mean the larger volume of sales, the larger would be the loss. In the event f under filling, the customers receives less than 300 ml of the drink at the time payment is made for 300 ml. This could results in causing bad name to the company. Therefore, the company desires neither of the situations. The company would like to test the hypothesis if the average content of the containers varies from 300 ml. A one tailed test can be expressed as

H_0 : $\mu = 300$ ml

H_1 : $\mu \neq 300$ ml

We can call these hypotheses as two tailed hypotheses.

The problem of overfilling of containers could be written as:

$H = \mu = 300$ ml

$H_1 = \mu > 300$ ml

These hypotheses are known as one tailed hypothesis. Here the analyst would be concerned in the upper tail of the sampling distribution. However, if the issue is under-filling of the containers, the hypotheses may be stated as:

$H_0 = \mu = 300$ ml

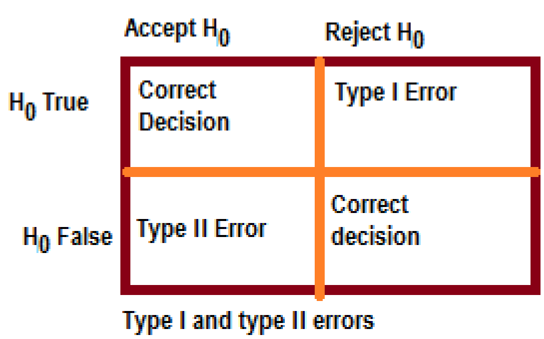
$H_1 = \mu < 300$ ml

is of the distribution

The researcher now has to reduce the descriptive and relational hypotheses to a statistical H_0 as well as the corresponding alternative hypotheses as H_1

6. **Type I and type II error**

The acceptance and rejection of a hypothesis depends upon the similar results. There is ever a chance of the sample not representing the population and therefore, resulting in errors and the results drawn could be faulty. The scenario could be as shown as in Fig. 12.1.



i

Figure 2 Type I and Type II error

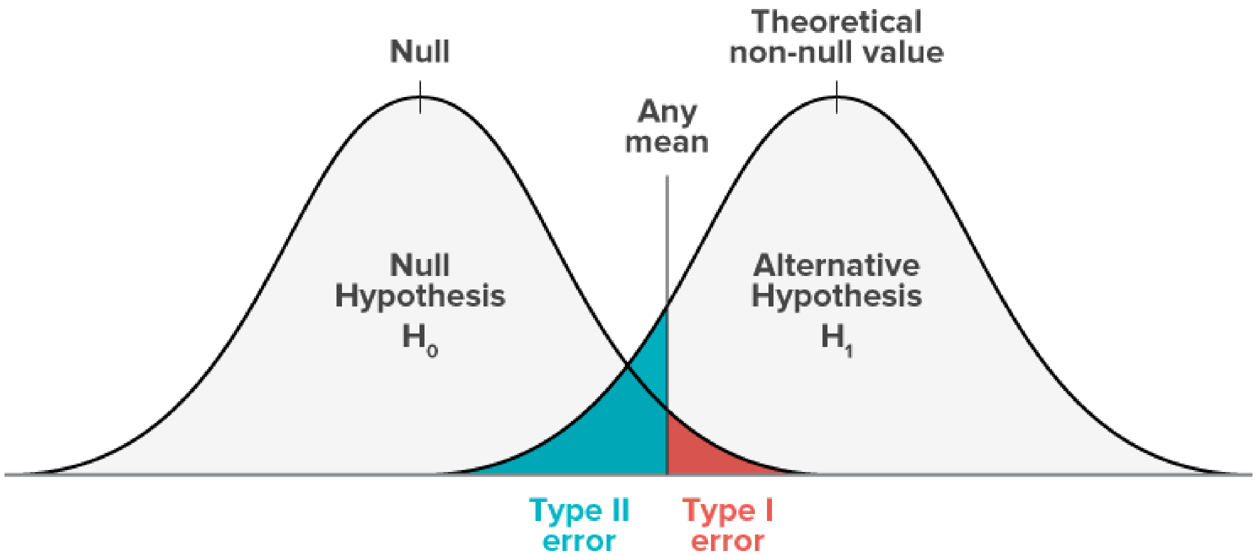


Figure 3 Type I and Type II Error

If null hypothesis is true and is accepted or when the null hypothesis is false is rejected, in either case the decision is right. However, if the hypothesis is actually true, but is rejected, Type I error is committed. The committing of Type I error is indicated with alpha (α). Similarly, if the null hypothesis is accepted, when actually it is false the researcher commits type II error. The chances of committing a Type II error are indicated by beta (β). $1-\beta$ is called the power of test.

7. Procedure of Testing of Hypothesis

In testing of a hypothesis, the following major steps are observed. These steps are shown in Fig.2.

4.1 Setting up of a hypothesis

The first step in hypothesis testing is to get a hypothesis about a population parameter. The conventional approach to hypothesis testing is not to construct a single hypothesis, but rather to set up two different hypotheses. State the null hypothesis, H_0, and the alternative hypothesis, H_a. The set up alternative hypothesis represents what the researcher is trying to prove. The null hypothesis represents the negation of what the researcher is trying to prove.

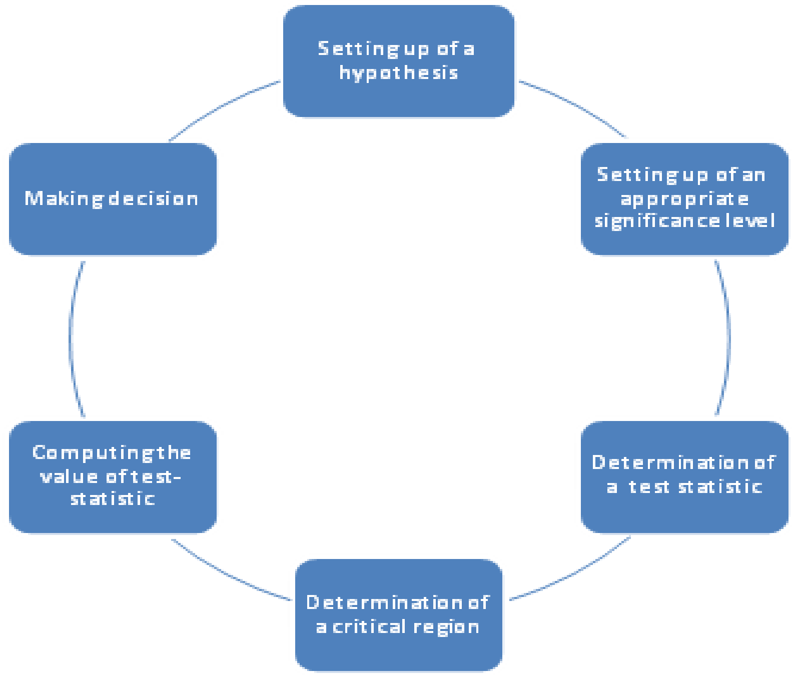


Fig.2 Steps in Testing of Hypothesis

4.2 Level of Significance

The function of hypothesis testing is compelled to choose a significance level. The level of significance (α) is chosen prior to sample selection. The significance level indicates the anticipation of rejection of the null hypothesis at the time it is really correct.

The value varies from issue to issue but often it is adopted as either 5 per cent or 1 percent. A five per cent level of significance is interpreted that there would be 5 chances out of one hundred that a null hypothesis is rejected while it is to be accepted. This means that the researcher is 95 percent confident that a right decision has been taken. Therefore it is seen that the confidence with which a researcher rejects or accepts a null hypothesis depends upon the level of significance. When the null hypothesis gets rejected at a particular level of significance, the result is thought to be significant. A hypothesis rejected at 1 per cent level of significant, must also be rejected at 5 per cent level of significance

4.3 Decision on a test statistics

The next stage in hypotheses testing procedure is to construct a test criterion involving selection of a suitable probability distribution for the specific test, i.e. a probability distribution that can appropriately be applied. Some probability distribution that are commonly used in testing procedures are t, F X^2. Test criteria must employ an appropriate probability distribution; for example, if only small sample is available, the use of the normal distribution would not be appropriate. Various test statistics include t, Z, X^2 or F, depending upon different assumptions.

4.4 Determination of a critical region

Prior to a sample from the population is drawn, it is crucial to clearly state the values of test statistic resulting in rejection or acceptance of the null hypothesis. The critical region causes the null hypothesis to be rejected. With a given a level of significance, α is the optimal critical region for a two-tailed test containing of α/2 per cent area in the right tail of the distribution plus α/2 per cent in the left tail of the distribution if the null hypothesis is rejected. Therefore, establishing a critical region is similar to determining a 100 (1-α) per cent confidence interval.

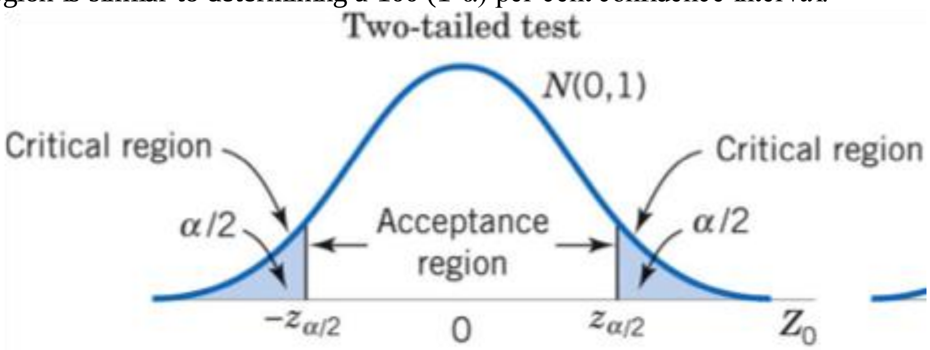


Figure 4 Critical Region

Adapted from cma.cmappers.net

4.5 Computing the value of test-statistics

The value of the test statistic is dependent upon the random sample of size n. Once the computation of the value of test statistic is carried out, we must verify if the sample results take place in the region of criticality or the acceptance region.

4.6 Decision-Making

There may be two alternatives. The hypothesis either is rejected or is accepted based upon the occurrence of the value of the test statistic in rejection region or the region of acceptance. The management decision relies upon the statistical decision for rejecting null hypothesis or accepting the null hypothesis.

If the analyst tests the hypothesis at 5 per cent level of significance, it would be rejected if the observed results have a probability less than 5 per cent. In such a case, the degree of contrast of the sample statistic and the population parameter is inferred to be significant. On the other hand, the acceptance of the hypothesis denotes that the deviation between the sample statistic and the population parameter is not significant. This difference may occur due to chance.

4.7 Standard Error and Sampling Distribution

The standard deviation of the sampling distribution is called the standard error. It measures the sampling variability due to chance or random forces. If a number of independent random samples of a particular size from a given population is selected and some statistic is calculated like the mean or standard deviation, from each sample, we get a series of values of these statistics. These values obtained from the different samples can be put in the form of a frequency distribution. The distribution so formed of all possible values of a statistic is called the sampling distribution or the probability distribution of that statistic. The symbols $\bar{X}$ and S are used to designate the mean and standard deviation of the sample distribution.

8. Test Statistic for Testing Hypothesis

In testing of hypothesis, it is important to apply a suitable test statistic. If the size of the sample is (n>30), Z statistic could be appropriate. For the small samples (n<30), when the standard deviation of the population is available z statistic can be used. However, if the standard deviation (δ) is not known and is assessed using sample data, a t test with suitable degree of freedom is applied under the presumption that the sample is drawn from a normal population.

Size of sample	Population Standard Deviation (δ)	
	Known	Not known
Large (n>30)	Z	Z
Small (n<30)	Z	t

9. Testing of Hypothesis

7.1 Test relating to Mean- Single Population

(a) Sample > 30

In case of large sample, or small sample if the population standard deviation are known Z test is applicable. There can be cases of two-tailed and one-tailed tests of hypothesis can occur

If Null hypothesis $H_0 : \mu = \mu_0$, Z test statistic can be used:

The test statistic is provided by:

$$Z = \frac{X - \mu Ho}{\frac{\sigma}{\sqrt{n}}}$$

Where

X= Sample mean

σ = Population standard deviation

μ_{Ho} = The value of μ under the assumption that the null hypothesis is true

n = Size of sample

In case the population standard deviation is not known, the sample standard deviation

$$s = \sqrt{1/n\text{-}1 \sum (X-\bar{X})^2}$$

is applied as an estimate of σ . It is important to note that $Z\alpha$ and $Z\alpha/_2$ respectively.

(b) Sample < 30

In case of small samples (n≤30) drawn from a population with a normal distribution and the population standard deviation is not known (α), a t test is applicable to perform the hypothesis for the test of mean. The t is a symmetrical distribution just like the normal one. However, t distribution occurrence is comparatively lower at the peak and higher at the tail. The t distribution is a more extent flat compared to the normal distribution. With the increase in the size of sample and the enhancement of the degrees of freedom, t distribution arrives the normal distribution when n > 30.

Testing of the hypothesis:

The procedure for testing of a mean is similar to the case of large sample. The test statistic used in this case is

$$t_{n-1} = \frac{\bar{X} - \mu Ho}{\hat{\sigma}/\bar{X}}$$

Where $\hat{\sigma}/x = s/\sqrt{n}$ (s = Sample standard deviation)

n-1 = degrees of freedom.

7.2 **Tests for Difference between Two Population Means**

(a) Samples > 30

In both cases if the sample size more than 30, Z test is applicable. The hypothesis to be tested may be written as

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

$$\mu_1 = \text{mean of population 1}$$
$$\mu_2 = \text{mean of population 2}$$

This is two-tailed test. The test statistic is:

$$Z = \frac{(\bar{X}1 - \bar{X}2 - (\mu 1 - \mu 2)Ho}{\sqrt{\frac{\sigma 1^2}{n1} + \frac{\sigma 2^2}{n2}}}$$

$\bar{X}_1$= Average of sample drawn from population 1
$\bar{X}_2$= Average of sample drawn from population 2
n_1 = sample size drawn from population 1
n_2 = sample size drawn from population 2

If σ_1 and σ_2 are not known, their estimates given by σ1 and σ2 are used.

$$\hat{\sigma}_1 = s_1 = \frac{\sqrt{1}}{n1-1} \sum_{i=1}^{n1} (X1i - \bar{X}1)^2$$

$$\hat{\sigma}_2 = s_2 = \sqrt{\frac{1}{n2-1}} \sum_{i=1}^{n2} (X2i - \bar{X}2)^2$$

The Z estimate for the problem under consideration can be computed using the formula and tallied with the table value to accept or reject the hypothesis.

(b) Sample < 30

In case both the samples are (< 30) and the population standard deviation is not known, the procedure to discuss the equality of two population means would not be applicale in the sense that a t test would be applicable under the assumptions:

(a) The two population variances are equal
(b) Two population variances are not equal

Population variances are equal

If the two population variances are equal, their respective unbiased estimates are also equal. The formula can be expressed as:

$$\sqrt{\frac{\hat{\sigma}1^2}{n1} + \frac{\hat{\sigma}2^2}{n2}} = \sqrt{\frac{\hat{\sigma}^2}{n1} + \frac{\hat{\sigma}^2}{n2}} = \sigma\sqrt{\frac{1}{n1} + \frac{1}{n2}}$$

(Making an assumption of $\hat{\sigma}_1^2 = \hat{\sigma}_2^2$)

To attain an estimate of σ^2, a weighted average of $s^2_1\, s^2_2$ is applicable where the weights are degrees of freedom for every sample. The weighted average is an accumulated estimate of σ^2. The following equation gives the estimate:

$$\hat{\sigma}^2 = \frac{(n1-1)s1^2(n2-1)s^2}{n1+n2-2}$$

The testing procedure is as under**:**

$H_0: \mu_1 = \mu_2 \quad \rightarrow \quad \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2 \quad \rightarrow \quad \mu_1 \neq \mu_2 = 0$

After the computation of t statistic from the sample data, its comparison is made with the table value at the chosen level of significance in order to arrive at the decision for acceptance or rejection of hypothesis.

(c) Case of Paired sample (dependent sample)

The samples are not independent of each. In case of dependent samples (paired sample), two observations are taken from each respondent one prior to administering a the treatment and the other after the treatment has been administered. For example, some customers may be questioned on their perception about a product and later on, a television commercial may be shown to them about the same product. After seeing the advertisement, they may again be questioned on their perception about the product. Such a sample is called dependent or paired sample because on the same respondent two observations are taken- one prior to treatment and the other after being subjected to treatment. The objective of doing this could be to examine whether that perception has undergone a change after the subjects viewed the advertisement, and if so, in what direction.

The use of dependent sample enables us to perform a more precise analysis as it allows the controlling of extraneous variables. The difference is that we convert the problem from two samples to a one-sample score obtained by the management trainees divided randomly into two equal sizes, one taught by each method. After obtaining the scores by two methods, the null hypothesis of average scores being equal by two methods is written as:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 = \mu_2$$

Let $\mu_d = \mu_1 - \mu_2$

Since the pair sample observations are taken, the hypothesis is converted to:

$$H_0 : \mu_d = 0$$
$$H_1 : \mu_d \neq 0$$

This means that we want to test that the average difference in score is zero against the alternative hypothesis that it is not so. Here, d denotes the difference in scores by two methods.

The test statistic in such a case,

$$t = d'/s\, \sqrt{n}$$

which follows a t distribution with n-1 degrees of freedom,

where d' = Mean of difference = $\sum di/n$

s= standard deviation of differences $= \frac{\sqrt{\sum(d-d)^2}}{n-1}$

n= number of paired observations concerned with the sample

For a particular significance level α, the estimated t statistic is tallied with the tabulated (critical) t with n-1 degrees of freedom to accept or reject the null hypothesis.

Summary

The major research objective of inferential statistics is testing of research hypothesis through statistical hypotheses. Following the sampling approach, a hypothesis is accepted or rejected on the basis of sampling information along. A test is called one-tailed only if the null hypothesis gets rejected when the value of the test statistic falls in acceptance region of the distribution.
The acceptance and rejection of a hypothesis depends upon the similar results. There is ever a chance of the sample not representing the population and therefore, resulting in errors and the results drawn could be faulty. In testing of a hypothesis, the following major steps are observed. These steps followed are the setting up of a hypothesis, deciding the level of significance, decision on a test statistics, the determination of a critical region, computing the value of test statistics and making-decision. The standard deviation of the sampling distribution is called the standard error. It measures the sampling variability due to chance or random forces. In testing of hypothesis, it is important to apply a suitable test statistic. If the size of the sample is $(n>30)$, Z statistic could be appropriate. For the small samples $(n<30)$, when the standard deviation of the population is available z statistic can be used. However, if the standard deviation (δ) is not known and is assessed using sample data, a t test with suitable degree of freedom is applied under the presumption that the sample is drawn from a normal population. In case of large sample, or small sample if the population standard deviation are known Z test is applicable. In case the population standard deviation is not known, the sample standard deviation is applicable as an estimate of σ . It is important to note that $Z\alpha$ and $Z\alpha/_2$ respectively. In case of small samples $(n \leq 30)$ drawn from a population with a normal distribution and the population standard deviation is not known (α), a t test is applicable to perform the hypothesis for the test of mean.
In case of tests for difference between two population means where the sample size more than 30, Z test is applicable. The Z estimate for the problem under consideration can be computed using the formula and tallied with the table value to accept or reject the hypothesis. In case both the samples are (< 30) and the population standard deviation is not known, the procedure to discuss the equality of two population means would not be applicable in the sense that a t test would be applicable. If the two population variances are equal, their respective unbiased estimates are also equal.
To attain an estimate of σ^2, a weighted average of $s^2{}_1\, s^2{}_2$ is applicable where the weights are degrees of freedom for every sample.
In case of dependent samples (paired sample), two observations are taken from each respondent one prior to administering a the treatment and the other after the treatment has been administered.